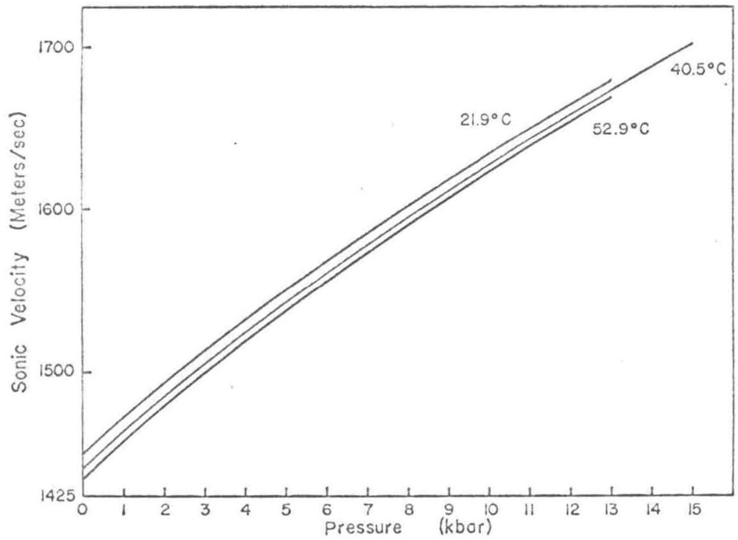

FIG. 4. The sonic velocity c_T in liquid Hg as a function of pressure at several temperatures.

T is absolute temperature, α is volume thermal-expansion coefficient, ρ is density, and C_P is specific heat at constant pressure. Since

$$\beta_T = -(1/V)(\partial V/\partial P)_T = (1/\rho)(\partial\rho/\partial P)_T, \quad (5)$$

where V is volume, it follows from Eq. (3) that

$$(\partial\rho/\partial P)_T = (1/c_T^2) + (T\alpha^2/C_P). \quad (6)$$

Therefore, on integration with respect to P,

$$\rho_{P,T} = \rho_{o,T} + \int_1^P \frac{1}{c_T^2}\,dP + T\int_1^P \frac{\alpha^2}{C_P}\,dP, \quad (7)$$

where $\rho_{o,T}$ is density at 1 atm and T, and $\rho_{P,T}$ is density at P and T. Evaluation of the two integrals over a range of T yields ρ as a function of P and T. The integral of $1/c_T^2$ at a given temperature can be evaluated directly from the least-squares velocity curves, using numerical integration, under the assumption that $1/c_T^2$ varies linearly for small $\Delta P = P_2 - P_1$. To evaluate the second integral, two additional relations can be used, viz.,

$$(\partial\alpha/\partial_P)_T = -(\partial\beta_T/\partial T)_P \quad (8)$$

and

$$(\partial C_P/\partial P)_T = -(T/\rho)[(\partial\alpha/\partial T)_P + \alpha^2]. \quad (9)$$

If β_T is known as a function of T at $P = P_1$, the initial slope of the α-vs-P curve may be determined for any temperature. Then, over the interval $\Delta P = P_2 - P_1$, the change of α can be approximated by

$$\alpha_P = (\partial\alpha/\partial_P)_{T,P_1}(P - P_1) + \alpha_{P_1}, \qquad P_1 \leq P \leq P_2. \quad (10)$$

If α vs T is known at P_1, α vs T at P_2 may be found from (10). Therefore, in (9), $(\partial\alpha/\partial T)_P$ can be expressed as

$$\left(\frac{\partial\alpha}{\partial T}\right)_P = \left[\frac{(\partial\alpha/\partial T)_{P_2} - (\partial\alpha/\partial T)_{P_1}}{\Delta P}\right](P - P_1) + \left(\frac{\partial\alpha}{\partial T}\right)_{P_1},$$
$$P_1 \leq P \leq P_2. \quad (11)$$

Using (11) and the square of (10), Eq. (9) may be integrated analytically to find ΔC_P if ρ is assumed constant over ΔP. This produces a negligible error if ΔP is small. On performing the integration, ΔC_P is found to be very small so that C_P may be assumed constant for the integration in (7). Only the integral of α^2 at each temperature needs then to be evaluated. This can be found by using the square of (10) and integrating analytically. The density at P_2, $\rho_{P_2,T}$ is therefore determined.

It is apparent that in the course of calculating $\rho_{P_2,T}$ both C_P and α have been determined as a function of T at P_2. Clearly, then, β_T may be calculated as a function of T at P_2 from Eq. (4). It follows that the calculation may be now repeated starting with the integration of $1/c_T^2$ over the interval $\Delta P = P_3 - P_2$ and the determination of $(\partial\alpha/\partial T)_P$ at P_2 from Eq. (8). By continual repetition of these calculations all the quantities β_T, α, C_P, ρ, and β_{ad} may be determined as a function of temperature and pressure. In the experiments described above the change of velocity with

TABLE IV. One-atmosphere input data to the compression calculation.

Temperature, t (°C)	21.9°	40.5°	52.9°
Density, ρ (g/cm³)[a]	13.54122	13.49573	13.46551
Thermal-expansion coefficient, α (°C⁻¹×10⁴)[b]	1.81069	1.80825	1.80699
Specific heat, C_P (ergs/g·deg×10⁻⁶)[c]	1.390	1.385	1.382
Sonic velocity c_T (m/sec)[d]	1450.1	1441.5	1435.8

[a] Reference 36.
[b] Reference 35.
[c] Reference 34.
[d] Reference 27.